**Coloring the square of a planar graph with girth restrictions**

A graph is $k$-colorable if each vertex receives a color from 1 to $k$ so that no vertex receives the same color with one of its neighbors. Hence, the maximum degree $∆G$ of a graph $G$ is an important quantity for chromatic graph theory. A simple greedy algorithm enables us to color a graph $G$ with $∆G+1$ colors. The well-known Brooks' theorem actually characterizes when we need all $∆G+1$ colors. The square of a graph $G$ is formed from $G$ by adding extra edges $uv$ whenever $u$ and $v$ have distance two in $G$. Since each closed neighborhood of a vertex forms a clique in the square of a graph $G$, the number of colors required must be at least $∆G+1$.

In fact, Wegner constructed a graph $G$ that requires roughly $1.5∆G$ colors, and moreover $G$ is planar! Sparked by a conjecture of Wang and Lih, researchers were able to prove that for each $g\geq 5$ there exist tight constants $D\_{g}$ and $C\_{g}$ such that if a planar graph $G$ has minimum cycle length at least $g$ and $∆G\geq g$ , then the square of $G$ is $∆G+C\_{g}$-colorable. Instead of asking for the threshold on the minimum cycle length, we extend the question and ask which cycle lengths must be forbidden in order to obtain a bound of the form maximum degree plus a constant. We completely solve this question when we consider the class of planar graphs with a forbidden set of cycle lengths. Namely, we prove the following: For a finite set $S$, there exists a constant $C\_{S}$ such that the square of a planar graph $G$ without cycle lengths in $S$ is $∆G+C\_{S}$-colorable if and only if $4\in S$.