**Coloring the square of a planar graph with girth restrictions**

A graph is -colorable if each vertex receives a color from 1 to so that no vertex receives the same color with one of its neighbors. Hence, the maximum degree of a graph is an important quantity for chromatic graph theory. A simple greedy algorithm enables us to color a graph with colors. The well-known Brooks' theorem actually characterizes when we need all colors. The square of a graph is formed from by adding extra edges whenever and have distance two in . Since each closed neighborhood of a vertex forms a clique in the square of a graph , the number of colors required must be at least .

In fact, Wegner constructed a graph that requires roughly colors, and moreover is planar! Sparked by a conjecture of Wang and Lih, researchers were able to prove that for each there exist tight constants and such that if a planar graph has minimum cycle length at least and , then the square of is -colorable. Instead of asking for the threshold on the minimum cycle length, we extend the question and ask which cycle lengths must be forbidden in order to obtain a bound of the form maximum degree plus a constant. We completely solve this question when we consider the class of planar graphs with a forbidden set of cycle lengths. Namely, we prove the following: For a finite set , there exists a constant such that the square of a planar graph without cycle lengths in is -colorable if and only if .