Design of a Real Option Contract

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Introduction

Many contracts in the real world take a form of options (i.e., the right, but not the obligation).

- Oil-gas leases: an upstream firm is granted the right to drill for and produce oil on the leased land.
- Joint venture agreements: each company is granted the right to abandon the given business.
- Standard operating leases: the lessee receives the right to cancel the lease at any time during the life of the contract.

INTRODUCTION

- After signing up, the contractor (agent) has to decide
  - when to execute the option
  - within a predetermined time period,

  considering the time-varying asset value.

- This stopping decision also affects the payoff of the other party (principal).

- The principal therefore has to design a contract so as to align the agent's incentives with hers.

"Oil and Gas Lease Utilization" - Report to the President

In March 2011, the Obama administration directed a study to determine how public lands leased to companies are being utilized, as part of a strategy to prod companies to produce more natural resources from existing leases.

	Onshore	**Offshore**
Total Leased Acres	37M (49,213 lease)	35.8M (6,621 lease)
Inactive Lease Acres	20.7M (21,906 lease)	25.7M (4,580 lease)
Active Lease Acres	16.3M (27,301 lease)	10.1M (2,041 lease)

Table: *Oil and Gas Lease Utilization-Onshore and Offshore*, May 2012, DOI

About 57% (70%) of the onshore (offshore) acreage under lease are idle, i.e. neither actively producing nor being part of a future production plan.

Obama moved forward on new oil and gas rules for public lands: 50% increase in royalty rates (March 2015).

What made companies hesitate in drilling?

- Drilling is an up-front investment in future production for several years.
- Once drilled, the cost for oil well casings cannot be recovered.

Figure: Crude Oil Prices Chart in 2009 $\sim$ 2012

Real option theory: as the expected volatility of the future oil price increases, the incentives to delay their investment would grow stronger. But, is that all? (particularly for the principal)

Basic Elements of Oil-Gas Lease

The change in the royalty rate itself would not be effective enough because there are other important things to consider.

The standard oil and gas lease contract in North America has three instruments to make companies' incentives be aligned with the principal's.

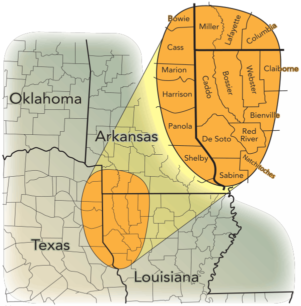
(1) Down payments (bonus): unconditional money paid to the principal, agreed upon both parties to be given on signing of the lease.

(2) Royalty payments: an agreed on percentage of the profits from marketing the oil and gas. The drilling costs are deducted and the principal receives a given percentage of the remainder.

(3) (primary) Term: the period of time during which the lease will be in effect. If a well was drilled within the term, then the lease remains in force so long as oil continues in "paying quantities". Otherwise, the lease will expire and the ownership is reverted to the principal.

Leases in the Haynesville Shale

- Royalties are typically between 19% and 25%.



Variables	Mean	P5	P50	P95
Term	37.2	36	36	60
Royalty	23	18.8	25	25
Extension	24.1	24	24	24

Table: *Summary statistics for leases*, HKL (2019)

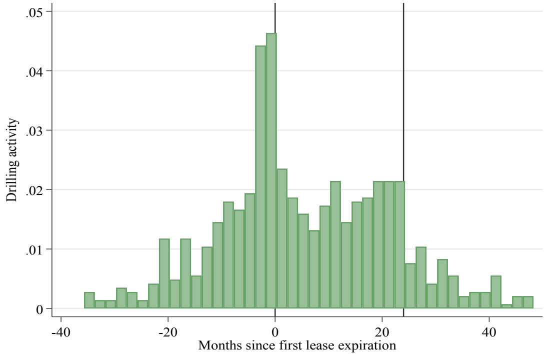
- Most leases have 36 months primary terms.
- About 78% of leases have extension clauses (require an additional bonus).

Royalty Rate

- The role of royalty is well-documented in literature: Riley (1988), Hendricks et al. (1993).
- It can serve to reduce the rents given up to the agent but also delays execution of the option.

Term

- What about the primary term?



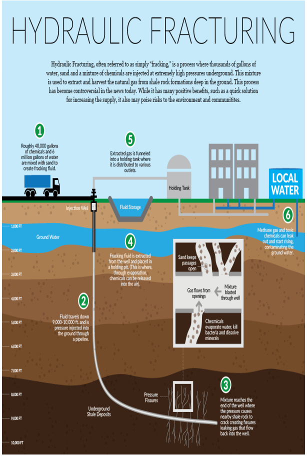
Note: Vertical lines are drawn at the date of first lease expiration and two years after first lease expiration. Bars are 3 months wide.

Figure: Timing of Drilling in Haynesville, Herrnstadt et al (2019)

- Secondary term?

Source of Information Asymmetry

Hydrofracking is a technique designed to recover oil and gas from shale rock, adopted in the US since 1950 and significantly boosting domestic oil production.



Its cost ranges from \$0.1M for a shallow well (5,000 ft) to \$8M for a very deep well (up to 20,000 ft).

The firm does not know in advance the exact drilling cost but it obtains better information by conducting seismic surveys than the principal.

Therefore, it is quite natural to assume that the drilling cost is the firm's private information.

Main Questions

(1) What is the optimal real option (lease) contract?

- What is the role of (optimal) term in the context of information asymmetry?

(2) Without commitment, how to design the renegotiation?

- What is the role of (optimal) term in the context of information asymmetry under the renegotiation case?

Our Main Findings: Full Commitment

Our screening model is simple but useful to understand why most oil wells remain inactive, incorporating information asymmetry into the standard real option framework.

The augmented model provides two important predictions:

(1) Complete Information: The principal can implement the perfectly aligned investment timing by a simple royalty payment rule independent of the firm's type.

(2) Private Information: The optimal contract features downward distortion and thus *delays* the inefficient firm's investment by extending the term (relative to the first-best term).

In this case, the principal can effectively reduce the efficient firm's information rents (the tradeoff between rent extraction and surplus loss).

OUR MAIN FINDINGS: RENEGOTIATION

When the renegotiation case is allowed, the principal tries to learn the type of the firm over time and the firm tries to hide it.

(1) The optimal strategy after the renegotiation time T_1 is the same as the commitment case. The principal suggests the revised contract based on her belief at T_1.

(2) The equilibrium is semi-separating in the sense that the efficient type will use the mixed strategy.

(3) There is a tradeoff, which determined optimal T_1.

- short-term: a little learning with small cost
- long-term: full learning with high cost

Literature and Our Approach

- Board (2007 JET) - government auction to determine (z,s) with the predetermined T

- Grenadier and Wang (2005 JFE) - real option contracting under adverse selection in an infinite horizon problem:
  - The exercise time is enforceable.

- Cong (2018 MS) - "Auction timing" problem with (z,s) with the infinite lease term, state-contingent royalty payment

- Ours: Adverse Selection (Screening) to determine (z,s,T) - role of the term
- The investment (drilling) timing is the firm's decision. The principal cannot directly enforce it although there is no hidden information.
  - The contract term affects the *time value* of the option.
  - We consider the renegotiation case.

An Illustrative Example

Consider an upstream company who decides whether to drill for oil on real estate. The land is owned by one individual who we shall call the principal hereafter.

The decision of drilling is completely irreversible—once drilled, the pump can only be used to produce oil. The firm has a technology of producing one barrel of oil every period.

Let X_t denote the unit price of oil in period t with initial price $X_0 = 200$. In period $t = 1$, the price is expected to rise to 300 with prob p or fall to 100 with prob $1 - p$.

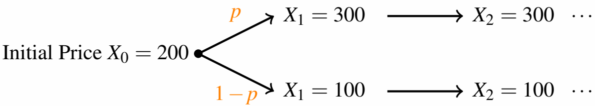


Figure: Price Uncertainty Lasting Two Periods

To keep our example as simple as possible, assume that after a change the price will remain at the new level forever.

Drilling cost is a binary random variable: $c \in \{c_L, c_H\}$ with $c_L < c_H$, which is the firm's private information. The principal believes $c = c_L$ with probability q.

Assume that the reservation utility of the firm is zero (Participation Constraint).

The principal offers the firm an option contract $O \equiv \{z,\ s,\ T\}$ that is comprised of

(1) bonus $z \in \Re_+$,

(2) royalty $s \in (0, 1)$ to the principal, and

(2) term $T = \{0, 1, \cdots, \infty\}$.

In this example, the firm's stopping decision τ with type $k = L, H$ will take a form of

- $\tau_k = 0$: drilling right away or
- $\tau_k = 1$: holding off making a decision to the next period, and drilling if $X_1 = 300$.

Principal's Problem: Screening

The principal offers a menu of contracts $\{O_L, O_H\}$

$$O_L = (z_L, s_L, T_L), \qquad O_H = (z_H, s_H, T_H)$$

in order to separate the different types. We write as $\Pi_k(\tau_k | O_m)$ the expected payoff of type-k firm when the firm chooses a option contract O_m and adopts the stopping decision τ_k.

As is well-known, the incentive compatibility condition for the efficient type (type L) is binding in the optimal contract:

$$\Pi_L(\tau_L^* | O_L) = \max_{\tau} \Pi_L(\tau | O_H).$$

The expression on the right-hand side

$$\max_{\tau} \Pi_L(\tau | O_H) = \max_{\tau} -z_H + \mathbb{E}\left[\mathbb{1}_{\{\tau \leq T_H\}}\left((1 - s_H)\sum_{t=0}^{\infty} \delta^t X_t - \delta^{\tau} c_L\right)\right]$$

captures the informational advantage of type L and determines its information rents.

Suppose $T_H = 0$. Then $\tau_H = 0$ (if s_H is not too high) and the down payment z_H is

$$z_H = (1 - s_H)\mathbb{E}\left[\sum_{t=0}^{\infty} \delta^t X_t\right] - c_H,$$

which can be obtained from $\Pi_H(\tau_H | O_H) = 0$. Note that facing the same contract, the type L is willing to execute the option earlier than the type H. Therefore, its information rents amount to

$$\begin{aligned}\max_{\tau} \Pi_L(\tau | O_H) &= -z_H + (1 - s_H)\mathbb{E}\left[\sum_{t=0}^{\infty} \delta^t X_t\right] - c_L \\ &= c_H - c_L := \Delta_c > 0.\end{aligned}$$

Suppose now $T_H = 1$. The type-H firm would delay drilling to period 1 if

$$\Pi_H(1|O_H) \geq \Pi_H(0|O_H) \;\; \Leftrightarrow \;\; c_H(1-p\delta) \geq (1-s_H)\left[200+(1-p)\cdot\frac{100\delta}{1-\delta}\right],$$

that is, if s_H is sufficiently high. Like the previous case, the corresponding down payment can be found from the binding participation constraint:

$$z_H = p\delta\left[(1-s_H)\frac{300}{1-\delta} - c_H\right].$$

In this case, the type L's information rents become

$$\Pi_L(\tau|O_H) = \begin{cases} (1-s_H)\left[200+(1-p)\cdot\frac{100\delta}{1-\delta}\right] - c_H(1-p\delta) + \Delta_c & \text{if } \tau = 0 \\ p\delta\Delta_c & \text{if } \tau = 1. \end{cases}$$

Either case, the rents would be smaller than Δ_c.

This example shows us that the principal can mitigate the incentive problem due to hidden types by extending $T_H = 1$ and setting s_H high enough.

- the surplus loss from implementing $\tau_H = 1$ rather than $\tau_H = 0$
- the expected gain from rent extraction, $q(1-p\delta)\Delta_c$.
- The latter is greater than the former: The principal can strictly benefit from delaying the inefficient type's investment decision.

Note: This example does not specify the principal's opportunity cost.

Model Setup

Suppose there is real estate owned by a principal and a company (agent) acquires a license to drill on the land.

The cash flow X_t is generated once the firm develops the well at t. X_t is $\mathscr{F}_t$-adapted process on a standard probability space on $(\Omega, \mathbb{P}, \mathscr{F})$ satisfying

$$\mathbb{E}_\tau\left[\int_\tau^\infty e^{-r(t-\tau)} X_t dt\right] < \infty,$$

where $\mathbb{E}_\tau[\cdot] = \mathbb{E}[\cdot|\mathscr{F}_\tau]$. WLOG, we assume that X_t *is the payoff at* t.

The drilling cost is hidden information:

$$c \in \{c_H, c_L\} \text{ with } (c_L < c_H) \text{ and } \mathbb{P}(c = c_L) = q.$$

The contract consists of (i) upfront fee $\{z_H, z_L\}$ (ii) royalty rate $\{s_H, s_L\}$ (iii) term $\{T_H, T_L\}$.

The license is terminated if the firm does not invest until T_i. The principal's expected reservation utility is $Y \geq 0$.

THE PRINCIPAL'S PROBLEM: SCREENING

The principal offers the menu of contract $\{O_L, O_H\} = \{(z_L, s_L, T_L), (z_H, s_H, T_H)\}$ to maximize her expected profit under the incentive compatibility conditions and the participation constraints.

$$\max_{\{O_L, O_H\}} q\left(z_L + \mathbb{E}\left[\mathbb{1}_{\{\tau_L \le T_L\}} e^{-r\tau_L} s_L X_\tau + \mathbb{1}_{\{\tau_L > T_L\}} e^{-rT_L} Y\right]\right)$$

$$+ (1-q)\left(z_H + \mathbb{E}\left[\mathbb{1}_{\{\tau_H \le T_H\}} e^{-r\tau_H} s_H X_\tau + \mathbb{1}_{\{\tau_H > T_H\}} e^{-rT_H} Y\right]\right)$$

subject to

$$\tau_k \in \underset{\tau}{\operatorname{argmax}} \underbrace{-z_k + \mathbb{E}\left[\mathbb{1}_{\{\tau_k \le T_k\}} e^{-r\tau}\left((1-s_k)X_\tau - c_k\right)\right]}_{\equiv \pi_k(\tau | O_k)} \qquad k = L, H \qquad (\mathrm{IC}_\tau)$$

$$\Pi_k \equiv \pi_k(\tau_k | O_k) \ge \max_{\tau} \pi_k(\tau | O_m), \qquad k, m \in \{L, H\} \text{ with } k \ne m \qquad (\mathrm{IC}_k)$$

and

$$\Pi_k \ge 0, \qquad k \in \{L, H\} \qquad (\mathrm{PC})$$

Notice that there are two types of ICs: adverse selection and moral hazard

Complete Information

If the types are observable, (PC) is biding. For the efficient type $c = c_L$, we have

$$z_L = \max_{\tau} \mathbb{E}\left[\mathbb{1}_{\{\tau \le T_L\}} e^{-r\tau}\left((1-s_L)X_\tau - c_L\right)\right]. \tag{1}$$

Substituting this into the principal's objective function, we have

$$\max_{O_L^F} \Phi_L(T_L) := \mathbb{E}\left[\mathbb{1}_{\{\tau \le T_L\}} e^{-r\tau}\left(X_\tau - c_L\right) + \mathbb{1}_{\{\tau > T_L\}} e^{-rT_L} Y\right], \tag{2}$$

while the firm's stopping decision given $\left(T_L, s_L\right)$ satisfies (1).

The royalty rate does not directly show up on the principal's payoff, but has an impact through the firm's optimal choice of τ.

IMPLEMENTABILITY

$$
\begin{aligned}
&\mathbb{E}\left[\mathbb{1}_{\{\tau \le T_L\}} e^{-r\tau}\left(X_\tau - c_L\right) + \mathbb{1}_{\{\tau > T_L\}} e^{-rT_L} Y\right] \\
&= \mathbb{E}\left[\mathbb{1}_{\{\tau \le T_L\}} e^{-r\tau}\left(X_\tau - c_L\right) - \mathbb{1}_{\{\tau \le T_L\}} e^{-rT_L} Y\right] + e^{-rT_L} Y \\
&= \mathbb{E}\left[\mathbb{1}_{\{\tau \le T_L\}} e^{-r\tau}\left(X_\tau - \left(c_L + Y e^{-r(T_L - \tau)}\right)\right)\right] + e^{-rT_L} Y
\end{aligned}
$$

PROPOSITION 4.1. *Suppose* (τ^*, T_L^f) *maximizes the principal's value in equation* (2)*. Then, there exists* $s_L^f \in (0,1)$ *such that* τ^* *is the optimal stopping time for the firm's problem of* (1) *with* s_L^f*.*

In fact, $s_i^f(t,x) = \frac{Y}{xe^{r(T-t)}}$ for each i.

TERM (T_H, T_L)

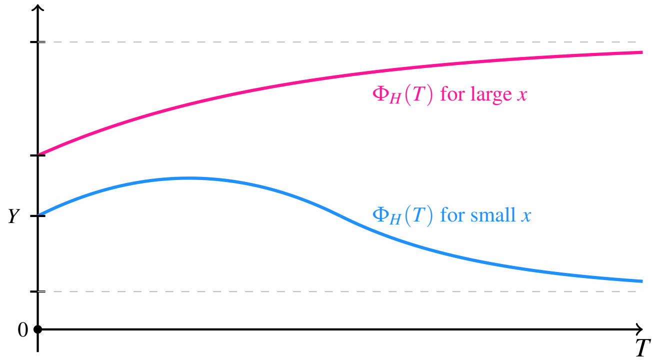
We are left to determine the terms (T_H, T_L).

If Y is sufficiently small (including $Y = 0$) or if $X_0 = x$ is sufficiently high, i.e., $Y << x$, then $\Phi_i(T)$ is monotonic increasing for $i = L, H$. In other words, $T_H = \infty$ and $T_L = \infty$.

Suppose $X_0 = x$ is not too high. There exists $T_i \in [0, \infty)$ for $i = L, H$ to maximizes the principal's objective function. Moreover,

- $T_H = T_L = 0$ only if the real options are initially deep in the money, i.e., the costs (c_H, c_L) are sufficiently lower than x.
- Otherwise, $T_H > T_L$

Principal's Value (Optimal Term)



Main Finding (Complete Information)

- If the cost is observable to both parties, the principal can implement the perfectly aligned investment time (even in the case of delegation) by using the royalty rate.

- The optimal royalty rate $s_i^f(t,x) = \frac{Y}{xe^{r(T-t)}}$ is simple:
  - The rate decreases with the exercise time: to provide incentives to early exercise.
  - The royalty rate is independent of the firm's cost.
  - If the reservation utility increases, it is optimal to increase the rate:
    - Obama vs Trumph (in terms of environment) if $Y = Y(x_0)$.

- The upfront fee is to be set by the participation constraint. In particular, if $Y = 0$, $s_i^f = 0$ and $z_i^f > 0$, which means "sell the option".

Incomplete Information

Suppose the principal cannot observe the cost type. Suppose (T_H, s_H) is given. Then, (PC) for the inefficient type is binding for some τ_H, i.e., we have

$$z_H^* = \mathbb{E}\left[\mathbb{1}_{\{\tau_H \le T_H\}} e^{-r\tau_H}\left((1-s_H)X_{\tau_H} - c_H\right)\right].$$

On the other hand, (IC_k) for the efficient type is binding. More specifically,

$$\begin{aligned} \pi_L(\tau_L \mid O_L) &= \max_{\tau} \pi_L(\tau \mid O_H) = -z_H^* + \mathbb{E}\left[\mathbb{1}_{\{\hat{\tau} \le T_H\}} e^{-r\hat{\tau}}\left((1-s_H)X_{\hat{\tau}} - c_L\right)\right] \\ &= \underbrace{\mathbb{E}\left[\mathbb{1}_{\{\hat{\tau}=\tau_H=0\}}\Delta_c\right] + \mathbb{E}\left[\mathbb{1}_{\{\hat{\tau}<\tau_H \le T_H\}} Q\right]}_{\text{Information Rent}} \end{aligned} \tag{3}$$

where $\hat{\tau}$ is the optimal stopping choice of the efficient in the above equation and

$$Q := e^{-r\hat{\tau}}\left((1-s_H)X_{\hat{\tau}} - c_L\right) - e^{-r\tau_H}\left((1-s_H)X_{\tau_H} - c_H\right)$$

and $\Delta_c = c_H - c_L$.

PROPOSITION 4.2.

- *If $T_H = 0$, then we have $\hat{\tau} = \tau_H = 0$ and thus the information rent is $\Delta_c = c_H - c_L$.*
- *If $T_H > 0$ and $\tau_H > 0$, then the information rent is strictly smaller than Δ_c.*

PROOF.

The first part is immediate since the second expectation part of (3) is zero. Suppose $T_H > 0$ and $\tau_H > 0$. Then, the first expectation part of (3) is zero and the second one is

$$\mathbb{E}\left[e^{-r\hat{\tau}}((1-s_H)X_{\hat{\tau}} - c_L)\right] - \mathbb{E}\left[e^{-r\tau_H}((1-s_H)X_{\tau_H} - c_H)\right] < c_H - c_L$$

because of the standard property of the American call option premium (decreasing and strictly convex with respect to the strike price). □

Notice that the above argument is general since the proof is true for any X_t and s_H.

Principal's Value when $T_H = 0$ (Optimal Contract)

Suppose $x >> c_H > c_L$ so that $\tau_H = \tau_L = 0$. (T_H^*, T_L^*) is irrelevant. Then, the principal's profit is

$$q(-\Delta_c + (x - c_L)) + (1-q)(x - c_H) = \underbrace{x - (qc_L + (1-q)c_H)}_{\text{expected profit}} - \underbrace{\Delta_c}_{\text{information rent}},$$

$\Longrightarrow$ static adverse selection problem.

Therefore, in order to focus on the case when there is the value of waiting investment, we consider the case where the option is at or out of the money at least for the inefficient type ($x < c_H$).

PRINCIPAL'S VALUE WHEN $T_H > 0$ (OPTIMAL CONTRACT)

Suppose $T_H > 0$ and $\tau_H > 0$. Let us define the information rent $R(T_H;\hat{\tau},\tau_H)$ by

$$R(T_H) = \mathbb{E}\left[\mathbb{1}_{\{\hat{\tau}\le T_H\}} e^{-r\hat{\tau}}((1-s_H)X_{\hat{\tau}} - c_L)\right] - \mathbb{E}\left[\mathbb{1}_{\{\tau_H\le T_H\}} e^{-r\tau_H}((1-s_H)X_{\tau_H} - c_H)\right],$$

where τ_H and $\hat{\tau}$ are the optimal stopping times given (s_H, T_H). Notice that

$$R(T;\hat{\tau},\tau_H) = -z_L^* + \max_{\tau} \mathbb{E}\left[e^{-r\tau}((1-s_L)X_\tau - c_L)\right].$$

Plugging this and the binding participation constraint for the inefficient type into the principal's profit function with (T_H, T_L), we rewrite the principal's objective function as follows:

$$\max_{O_L, O_H} q\Phi_L(T_L) - qR(T_H;\hat{\tau},\tau_H) + (1-q)\Phi_H(T_H) \tag{4}$$

where

$$\Phi_i(T) := \mathbb{E}\left[\mathbb{1}_{\{\tau\le T\}} e^{-r\tau}(X_\tau - c_i) + \mathbb{1}_{\{\tau > T\}} e^{-rT} Y\right].$$

Rewrite (4) as

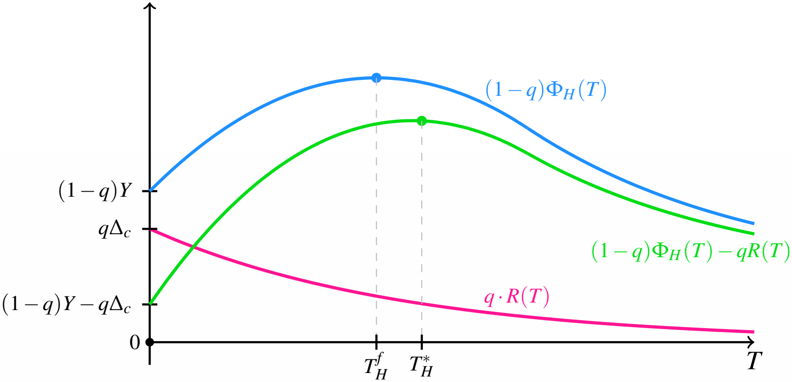
$$\max_{T_L} q\Phi_L(T_L) + \max_{(T_H, s_H)} -qR(T_H : \hat{\tau}, \tau_H) + (1-q)\Phi_H(T_H)$$

- The problem is decomposed into two independent parts.
- The first maximization problem is the same as the first-best case.
- In the second problem, the information rent is decreasing in T_H, while $\Phi_H(T)$ attains the maximum at $T = T_H^f$.

THEOREM 4.1.

(a) *The primary term for the efficient type is equal to the first-best primary term:* $T_L^* = T_L^f$. *Moreover, we have* $\tau_L = \tau_L^f$ *and* $z_L^* < z_L^f$.

(b) *The primary term for the inefficient type is great than the first-best primary term:* $T_H^* > T_H^f$. *In addition, we have* $\tau_H^* > \tau_H^f$


$(1-q)\Phi_H(T)$
$(1-q)\Phi_H(T)-qR(T)$
$q \cdot R(T)$
$(1-q)Y$
$q\Delta_c$
$(1-q)Y-q\Delta_c$
0
T_H^f
T_H^*
T

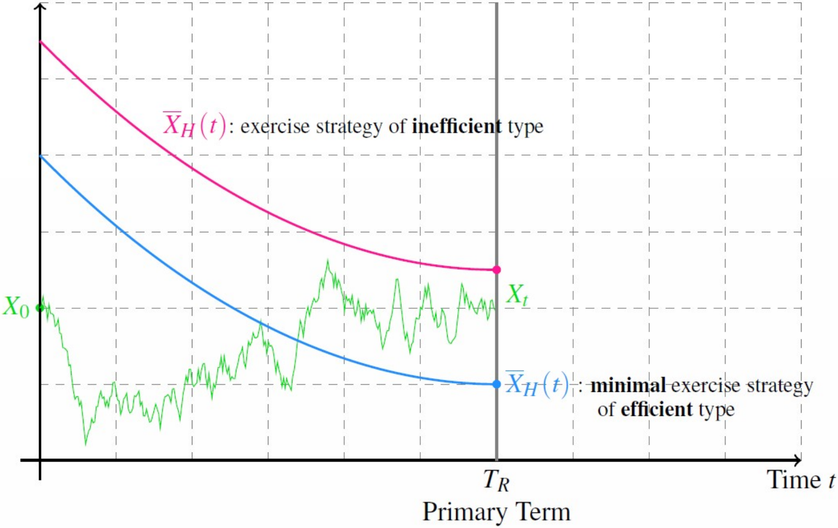
New Findings (Incomplete Information)

- If the cost is the private information, the optimal contract features downward distortion and thus *delays* the inefficient firm's investment by extending the term ($T_H^* > T_H$).
- There is no delay for the efficient firm with $s_L^* = s_L^f$.
- This result comes from the tradeoff between the rent extraction and surplus loss.
- Increasing the option value for the inefficient type decreases the information rent (by extending the term for the inefficient type) while it decreases surplus.
- $\tau_H^f < \tau_H^*$ and $s_H^f > ? < s_H^*$

Renegotiation Contract

- Normally the lease contract has a condition for renegotiation.
- What is the optimal contract?
- In particular, what is the role of the (optimal) term?
- Key Idea: Value from Learning versus Option Cost

SEMI-SEPARATING EQUILIBRIUM



CONCLUDING REMARKS: SUMMARY

- We investigate how to design the real option contract.
- The stopping decision affects the payoff of the principal, which implies that the principal has to design a contract in order to align the agent's incentives.
- (z, s, T).
  - Ours is the first paper to endogenously determine the optimal term in the contracting framework.
  - The role of the term is to mitigate the information problem by creating the option value of the inefficient type.
- Notice that the (private) value of the asset is fixed in the conventional bilateral trade under hidden information.
  - In the real option contract, the value of the asset (= intrinsic value + time value) is endogenously determined by the contract and the time value can be increased in order to mitigate the information rent, while there is surplus loss that comes from waiting longer.
- We also endogenize the renegotiation term (semi-separating equilibrium).

Concluding Remarks: Future Research

- Comparative statics analysis: volatility, growth rate, etc.
- Auction model to endogenize (z, s, T)
- Public land vs Private land
- Public (diversified) firm vs Private (undiversified) firm