Testing of Purchasing Power Parity Theory Using the Doubly Truncated ARMA-GARCH Model and MCMC Algorithms

Suduk Kim, Ajou University, Korea

Chyong Ling Chen, Feng Chia University, Taiwan

Hiroki Tsurumi, Rutgers University

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Abstract

Using a Markov Chain Monte Carlo algorithm, we show that the exchange rates of the Baht and of the Won are explained by the purchasing power parity theory when we incorporate the facts that these currencies were doubly truncated.

Key Words

doubly truncated regression model, Markov Chain Monte Carlo algorithms, highest posterior density interval, purchasing power parity

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1 Introduction

Around the time the Bretton Woods system began to collapse in early 1973, the foreign exchange rates of many countries began to be managed within upper and lower bounds. For example from 1972 to 1975 the European Economic Community (EEC) adopted the “snake-in-the-tunnel,” and the European Monetary System (EMS) was started in early 1979. In Japan the yen was floated in the early 1973 but it had been tightly controlled until the end of 1980. While by the mid 1980’s, the EEC countries and Japan had relaxed the foreign exchange controls moving more and more towards a free market, many developing countries continued to manage their currencies tightly pegging them to the U.S. dollar. Examples are Thailand and Korea, the two countries that are often identified as the major epicenters of the Asian financial crisis. Many who have studied the Asian financial crisis cite that the pegged exchange rate systems as a major cause of the financial crisis.

Under a pegged exchange rate system the foreign exchange rate fluctuates within certain bounds. If one wishes to test the purchasing power parity theory (PPP) using data on the foreign exchange rate of a country that has experienced the pegged system, one needs to incorporate information on currency control. In this paper we test PPP for the Baht and Won exchange rates. We use the doubly truncated regression model with an ARMA-GARCH error process and estimate them by Markov chain Monte Carlo (MCMC) algorithms. We find that PPP holds for these currencies when we incorporate the fact that they were under the pegged exchange rate systems.

The organization of the paper is as follows. In section 2 we present the regression models of testing PPP, and we explain how we introduce the bounds on the exchange rates. In section 3 we present numerical examples to highlight the Bayesian testing procedures. The estimated results are presented in section 4, and concluding remarks are given in section 5.
2 Models for Testing PPP

Purchasing power parity has been accepted as a central building block in the monetary models of exchange rate determination. PPP is based on an self-evident proposition that the exchange rate of two currencies will be determined by the ratio of aggregate prices of the two countries if market arbitrage enforces broad parity in prices across a wide range of goods and services. The existence of market arbitrage assumes that market mechanism is working.

The basic relationship between the nominal exchange rate and relative prices is

\[ E_t = \kappa \left( \frac{P_t}{P_t^*} \right)^\gamma \]  

where \( E_t \) is the nominal exchange rate of domestic currency, \( P_t \) and \( P_t^* \) are respectively the price indices of domestic and foreign countries. If \( \gamma = 1 \) then the nominal exchange rate is proportional to the price ratio, showing that PPP holds.

Surveying the literature we see that PPP has been tested using one of the following three models. Let us label them Model I, II, and III.

**Model I:** In equation (1) we set \( \gamma = 1 \) and transform it into the real exchange rate \( r_t = E_t P_t^*/P_t \):

\[ r_t = \kappa. \]

It is argued that if PPP holds in the long run then the real exchange rate \( r_t \) should follow a stationary process. Hence PPP is tested whether \( r_t \) has a unit root by the regression equation

Model I: \[ r_t = \mu + \rho r_{t-1} + \epsilon_t. \]

If \( \rho \geq 1 \) then \( r_t \) is a nonstationary process and PPP does not hold.

**Model II and III:** We test whether \( \gamma = 1 \) or not. To do so we put equation (1) into the following two regression models:

**Model II:** \[ \ln E_t = \gamma_1 + \gamma_2 (\ln P_t - \ln P_t^*) + u_t \]
Model III:  \[ \ln E_t = \gamma_1 + \gamma_2 \ln P_t + \gamma_3 \ln P^*_t + u_t \]

In model II we test \( \gamma_2 = 1 \) whereas in Model III we test the simultaneous restriction on the coefficients: \((\gamma_2, \gamma_3) = (1, -1)\). In models II and III, one may also test whether a cointegrating relationship holds among the regresand and regressors, although cointegration is not a part of the hypothesis test of PPP in Models II and III.

In the literature PPP has been tested by one of these three models using either the wholesale price index (WPI) or consumer price index (CPI). Table 1 presents a summary of the most recent literature on testing the PPP. Since the literature before 1995 has been surveyed elsewhere (Rogoff (1996) among others), the survey presents the studies that have been published since then, highlighting data, years under study, base country, price indices, models, test methods and findings. The findings in Table 1 are mixed, but we notice that in the recent years more and more of the results find support for PPP.

Suppose that the nominal exchange rate, \( E_t \), moves within the upper and lower bound

\[ a_t \leq E_t \leq b_t \]

where \( a_t \) and \( b_t \) are lower and upper bounds, respectively. The bounded \( E_t \) is illustrated in Figures 1 and 2. In Figure 1 the monthly observations on the exchange rate of the Thai currency per the U.S. dollar, \( E_t = \text{Baht/US} \) from January 1973 (1973.01) to February 1999 (1999.02) are plotted. There are three periods of \( E_t \): 1973.01–1984.11, 1984.12–1997.07, and 1997.08–1999.02. The first period (1973.01–1984.11) is the fixed exchange rate period with occasional realignment. The second period (1984.12–1987.07) is when the Baht was pegged to the basket of currencies, and the third period (1997.08–1999.02) is when the currency was forced to be floated. In Figure 1 the lower and upper bounds, \( a_t \) and \( b_t \) respectively, are drawn in for the second period. In Figure 2 the monthly observations on the exchange rate of the Korean currency per the U.S. dollar, \( E_t = \text{Won/US} \) from 1973.01 to 1999.02 are plotted. There are two periods of \( E_t \): 1973.01–1997.10, and 1997.11–1999.02. In the first period the Won was pegged to the U.S. dollar. The lower and upper bounds, \( a_t \) and \( b_t \), respectively, are drawn in for the first period.

The bounds on the Baht and Won exchange rates in Figures 1 and 2 are
### Table 1: Tests of PPP: A Brief Survey of the Literature

<table>
<thead>
<tr>
<th>Authors</th>
<th>Data</th>
<th>Time period</th>
<th>Exchange rates</th>
<th>Price index</th>
<th>Model</th>
<th>Test methods</th>
<th>Conclusions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hoque (1995)</td>
<td>Quarterly</td>
<td>1961–1990</td>
<td>Four Asian</td>
<td>WPI</td>
<td>2</td>
<td>t-test on $\beta$</td>
<td>$\beta$ coefficient estimates are greater than 1</td>
</tr>
<tr>
<td>Crownover et al. (1996)</td>
<td>Annual</td>
<td>1927–1992</td>
<td>6 countries</td>
<td>Constructed prices</td>
<td>2</td>
<td>ADF, t-test on $\beta$</td>
<td>PPP is supported</td>
</tr>
<tr>
<td>Leithian (1997)</td>
<td>Annual</td>
<td>1974–1990</td>
<td>22 OECD countries</td>
<td>CPI or CLI</td>
<td>1</td>
<td>unit root test for panel data</td>
<td>PPP is supported</td>
</tr>
<tr>
<td>Mclellan et al. (1997)</td>
<td>Monthly</td>
<td>1973–1990</td>
<td>9 countries</td>
<td>PPI</td>
<td>1</td>
<td>Bayesian test, ADF</td>
<td>Mixed results</td>
</tr>
<tr>
<td>Wu, Chen (1999)</td>
<td>Monthly</td>
<td>1981–1993</td>
<td>Pacific basin countries</td>
<td>CPI</td>
<td>1</td>
<td>Johansen</td>
<td>PPP is supported</td>
</tr>
<tr>
<td>Kakkar, Ogaki (1999)</td>
<td>Quarterly/Annual</td>
<td>various periods</td>
<td>US, Japan, UK, Italy, Canada</td>
<td>WPI, CPI tradable index</td>
<td>2</td>
<td>Johansen</td>
<td>PPP generally holds for longer-term periods</td>
</tr>
</tbody>
</table>

Notes: FARIMA=fractionally integrated ARMA model, CPI=consumer price index, WPI=wholesale price index, PPI=producer price index, CLI=cost of living index, ADF=augmented Dickey-Fuller test, HW=Horvath-Watson procedure, KPSS=Kwiatowski et al. test
estimated based on the chronology of foreign exchange rate management in
these countries given in Table 2.

We notice in Table 2 that the bounds of the Baht exchange rates were
undisclosed, and we shall explain later how we estimated the bounds.

If we ignore the fact that the exchange rate moved between the lower
and upper bounds, then testing of the PPP may tend to be biased. All the
studies in Table I treat the exchange rates as if they were unbounded.

Among the three models for testing PPP, let us choose Model II:

\[ y_t = \gamma_1 + \gamma_2 x_{t2} + u_t \]  

where \( y_t = \ln E_t \) and \( x_{t2} = \ln P_t - \ln P_t^* \). We use Model II because of its
simplicity and because we can reduce the PPP test to the test of a regression
coefficient.\footnote{In Models II and III we can test PPP by regression coefficients and at the same
time we can see whether the regression error term follows a unit root process. In the MCMC
algorithms it is easy to test the simultaneous hypothesis in Model III as a test on one
parameter. This can be done by deriving the posterior pdf for \( \theta = (\gamma_2 - 1)^2 + (\gamma_3 + 1)^2 \)
and by examining whether the posterior pdf contains \( \theta = 0 \).}

The foreign exchange rates exhibit volatility that may be captured by
a GARCH process, and thus let us specify an ARMA(\( p, q \))–GARCH(\( r, s \))
regression model. Hence, the error term in equation (2) is specified as

\[ u_t = \sum_{j=1}^{p} \phi_j u_{t-j} + \epsilon_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j} \]

\[ \sigma^2_t = \alpha_0 + \sum_{j=1}^{r} \alpha_j \epsilon_{t-j}^2 + \sum_{j=1}^{s} \beta_j \sigma^2_{t-j}, \]

\[ \alpha_0 > 0, \quad \alpha_j \geq 0, \quad j = 1, \ldots, r, \]

\[ \beta_j \geq 0, \quad j = 1, \ldots, s, \]

and \( \epsilon_t \sim N(0, \sigma^2) \).

Since \( y_t \) are doubly truncated for some sub-sample periods, let us use
the MCMC algorithms for the doubly-truncated ARMA(\( p, q \))–GARCH(\( r, s \))
regression model given in Goldman and Tsurumi (2001) to estimate the
Table 2: Exchange Rate Systems in Thailand and Korea, 1973–1997

<table>
<thead>
<tr>
<th>Thai Exchange Rate System</th>
<th>Korean Exchange Rate System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed exchange rate system with the daily rate determined by the commercial banks and the Exchange Equalization Fund. The Baht is pegged to the U.S. dollar at 23 Baht/US$ in 1981.</td>
<td>Market average of foreign exchange rate is adopted. The Won/US$ was pegged within the band of 2.25%.</td>
</tr>
<tr>
<td><strong>1984–1997</strong></td>
<td><strong>Nov. 11 1997</strong></td>
</tr>
<tr>
<td>The Baht was depreciated by 14.8% in November 1984, and it was pegged to the basket of currencies. Although the composition of the basket was kept secret but it was widely estimated that 80% of it was the U.S. dollar, 13% the Yen, and 7% the German Mark. The market average of foreign exchange rate was calculated. The exchange rate per U.S. dollar was announced everyday and was allowed to be within an undisclosed band.</td>
<td>The band was widened to 10%.</td>
</tr>
<tr>
<td><strong>July 2 1997</strong></td>
<td><strong>Dec. 16 1997</strong></td>
</tr>
<tr>
<td>A floating exchange rate system was adopted. The Baht depreciated 20% to 28.8 per U.S. dollar on this day.</td>
<td>A floating exchange rate system was adopted.</td>
</tr>
</tbody>
</table>
(2+p + q + r + s) parameters. In Appendix we present a general description of how to carry out the MCMC algorithms. In real data analysis, we normally do not know the orders of ARMA(p, q)–GARCH(r, s) and thus we need to determine these orders. In this paper we use a modified Bayesian information criterion as well as on the acceptance rates of the Metropolis-Hastings algorithms.

3 Numerical Examples

Let us illustrate how the posterior pdf’s of the regression coefficient of interest are affected by whether we ignore truncation or not. We generate \( \{y_t\} \) from the following ARMA(1,1)-GARCH(1,1) model:

\[
\begin{align*}
    y_t &= 1.0 + 1.0x_t + u_t \\
    u_t &= .9u_{t-1} + \epsilon_t + .3\epsilon_{t-1} \\
    \sigma_t^2 &= .5 + .3\epsilon_{t-1}^2 + .05\sigma_{t-1}^2 \\
    x_t &= 1.0 + .8x_{t-1} + v_t, \ x_0 = 10.0
\end{align*}
\]

and

\[\epsilon_t \sim N(0, \sigma_t^2), \quad v_t \sim \text{Uniform}(0, 1).\]

The sample size is 500. The upper and lower bounds are

\[a_t \leq y_t \leq b_t, \quad t = 1, \cdots, 500.\]  

We set two types of bounds: level truncation and change truncation that are defined as:

**Level Truncation**

\[a_t = a^*, \quad b_t = b^*\]  

where \(a^* = 8\), and \(b^* = 9\).
Change Truncation

\[ a_t = (1 - r_\ast)y_{t-1}, \quad b_t = (1 + r_\ast)y_{t-1} \]  
(7)

where \( r_\ast = .05 \) and \( r^\ast = .08 \). The model in (4)–(7) is designed to mimic foreign exchange rates that are managed to stay within fixed levels \( a_\ast \) and \( b_\ast \) or within the fixed percentage changes \( r_\ast \) and \( r^\ast \).

In reality we often do not know the lower and upper truncations. Hence, given data, \( \{y_t\}_{t=1}^{500} \), we estimate them as follows:

**Estimated Level truncation**

\[ \hat{a}_\ast = \min_{1 < t \leq 500} y_t = 8.001, \quad \hat{b}_\ast = \max_{1 < t \leq 500} y_t = 8.998 \]  
(8)

and

**Estimated Change truncation**

\[ \hat{r}_\ast = \min_{1 < t \leq 500} \left\{ \frac{\Delta y_t}{y_{t-1}} \right\} = .0499, \quad \hat{r}^\ast = \max_{1 < t \leq 500} \left\{ \frac{\Delta y_t}{y_{t-1}} \right\} = .0795 \]  
(9)

Figures 3a and 4a present the lower bounds, \( \{a_t\} \), truncated series, \( \{y_t\} \), and upper bounds, \( \{b_t\} \) for the level and change truncations, respectively. Figures 3b and 4b present the posterior probability densities of the slope coefficient, \( \gamma_2 \). The posterior pdf’s that are leveled “untruncated” are the posterior pdf’s that are obtained by treating \( \{y_t\} \) as if they are untruncated, whereas the posterior pdf’s that are leveled “truncated” are the posterior pdf’s that are obtained by incorporating the truncation into the likelihood function. We observe that the posterior pdf’s that ignore truncation lie clearly to left and their 95% highest posterior density intervals (HPDI’s) do not include \( \gamma_2 = 1 \). If we use bias as the generic term then the posterior pdf’s that ignore truncation are downward biased while the standard deviations are much smaller than those of the truncated cases.
Figure 1: Bhat/US$ Exchange Rates: 1973.01--1999.02

- Upper bound
- $y_t$
- Lower bound

X-axis: Year-Month
Y-axis: Bhat/US$
Figure 2: Won/US$ Exchange Rates: 1973.01--1999.02

Graph showing the exchange rates with upper bound, $\gamma_t$, and lower bound lines.

The exchange rate is pegged to the U.S. dollar until 1985 and then becomes floating.

Year-Month

Won/US$

73.01 85.06 89.08 93.10 97.12

pegged to the U.S. dollar

floating rate
Figure 3a: Truncated time series $y_t$ (n=500, level truncation)

Figure 3b: Posterior Pdf's of $\gamma_2$
Figure 4a: Truncated time series $y_t$ (n=500, change truncation)

Figure 4b: Posterior Pdf’s of $\gamma_2$
Let us test the PPP using Model II for the Baht/US$ and the Won/US$ exchange rates. We use the wholesale price indices (WPI) as well as the consumer price indices (CPI) as the price variables. As shown in Table 2 and Figure 2, the Baht exchange rate was fixed until November 1984 (1984.11), and thus we use monthly data from December 1984 (1984.12) to February 1999 (1999.02). Between November 1994 and June 1997, the Baht was pegged mainly to the U.S. dollar. The Baht/US$ rate was announced daily and was allowed to be within an undisclosed band. We estimated the lower and upper bounds of the change truncation as explained in the previous section:

\[
\begin{align*}
\hat{r}_s &= \min \left\{ \frac{\Delta y_t}{y_t-1} \right\} = 0.0227, \\
\hat{r}^* &= \max \left\{ \frac{\Delta y_t}{y_t-1} \right\} = 0.0224 \\
&\text{for } 1984.12-1997.06 \quad (10)
\end{align*}
\]

and we lift the bounds for 1997.07–1999.02.

In the case of the Won exchange rate, the Won/US$ was pegged within the band of 2.25% between March 1973 and October 1997, and the band was widened to 10% in November 1997. In early December of 1997, a floating exchange rate system was adopted. Hence, we lift the bounds for 1997.12–1999.02.

Since we are testing whether $\gamma_2 = 1$ or not using the highest posterior density interval (HPDI), let us present the posterior densities of $\gamma_2$ in Figures 5–8. The 95% HPDI’s are presented in Table 3.
Figure 5: Posterior Pdf's of $\gamma_2$: Bhat/US$ using WPI

untruncated:
mean: $-0.2227$
std: $0.1899$
95% HPDI: $(-0.47, -0.12)$

truncated:
mean: $0.8967$
std: $0.2505$
95% HPDI: $(0.41, 1.37)$
Figure 6: Posterior Pdf's of $\tau_2$ Bhat/US$ using CPI

Untruncated

- Mean: 1.890
- SD: 0.3460
- 95% HPDI: (-0.28, 0.78)

Truncated

- Mean: 1.2933
- SD: 0.5801
- 95% HPDI: (0.54, 1.81)
Figure 7: Posterior Pdf’s of $\gamma_2$: Won/US$ using WPI

- Untruncated:
  - Mean: 0.7998
  - SD: 0.1632
  - 95% HPDI: (0.75, 0.87)

- Truncated:
  - Mean: 0.7560
  - SD: 0.3532
  - 95% HPDI: (0.28, 1.21)
Figure 8: Posterior Pdf's of $\gamma_2$, Won/US, using CPI

- Truncated case
  - Mean: 0.8233
  - SD: 0.3920
  - 95% HPDI: (0.39, 1.40)

- Untruncated case
  - Mean: 0.7904
  - SD: 0.1577
  - 95% HPDI: (0.71, 0.88)
Table 3: 95% Highest Posterior Density Intervals of $\gamma_2$

<table>
<thead>
<tr>
<th>Exchange Rate</th>
<th>Untruncated</th>
<th>Truncated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baht/US$ Using WPI</td>
<td>$(-.47, -.12)$</td>
<td>$(.41, 1.37)^*$</td>
</tr>
<tr>
<td>Baht/US$ Using CPI</td>
<td>$(-.28, .78)$</td>
<td>$(.54, 1.81)^*$</td>
</tr>
<tr>
<td>Won/US$ Using WPI</td>
<td>$(.75, .87)$</td>
<td>$(.28, 1.21)^*$</td>
</tr>
<tr>
<td>Won/US$ Using CPI</td>
<td>$(.71, .88)$</td>
<td>$(.39, 1.40)^*$</td>
</tr>
</tbody>
</table>

*95% highest posterior density interval contains $\gamma_2 = 1$.

From Figures 5–8 and Table 3 we observe

1. The 95% HPDI’s contain $\gamma_2 = 1$ when we use the truncated model, whereas they do not contain it when we use the untruncated model regardless of whether the WPI or CPI is used.

2. The untruncated model for the Baht/US$ yields the posterior pdf’s of $\gamma_2$ with smaller means and standard errors than those for the truncated model.

3. For the Won/US$, the posterior pdf’s of $\gamma_2$ for the untruncated model have the means that are similar to those of the truncated model, but the standard errors of the untruncated model are much smaller than those of the truncated model. Due to the larger standard errors, the 95% HPDI’s of the posterior pdf’s using the truncated model contain $\gamma_2 = 1$.

4. The MCMC algorithms allow us to obtain the posterior distribution of

$$\rho = \max_{i=1,\ldots,p} \{\text{abs}(\text{root}_i)\},$$

where $\text{root}_i$ = inverse of the $i$-th root of $\Phi(B)$. Except for the Baht/US$ using CPI, all other cases show that the AR processes are stationary.

Tables 4–7 present the posterior summaries of the ARMA-GARCH models. In all cases GARCH error processes seem to explain the exchange rate models. As discussed in the Appendix the model selection is based on the modified Bayesian information criterion as well as the acceptance rates of the Metropolis-Hastings algorithms.
Table 4: Posterior Means and Standard Deviations: Baht/US$, Using WPI

<table>
<thead>
<tr>
<th></th>
<th>Untruncated ARMA(2,1)–GARCH(1,1)</th>
<th>Doubly Truncated ARMA(1,1)–GARCH(1,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_1 )</td>
<td>3.2533 (.0261)</td>
<td>3.3846 (.0430)</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>(-.2227) (.1889)</td>
<td>(.8967) (.2505)</td>
</tr>
<tr>
<td>( \phi_1 )</td>
<td>1.2131 (.1833)</td>
<td>(.8393) (.0920)</td>
</tr>
<tr>
<td>( \phi_2 )</td>
<td>(-.3249) (.1899)</td>
<td></td>
</tr>
<tr>
<td>( \rho )</td>
<td>(.8039) (.1252)</td>
<td>(.8393) (.0920)</td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>(.8167) (.1564)</td>
<td>(.1562) (.2458)</td>
</tr>
<tr>
<td>( \alpha_0 )</td>
<td>(.00078) (.0065)</td>
<td>(.00099) (.0057)</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>(.8393) (.0983)</td>
<td>(.4801) (.2069)</td>
</tr>
<tr>
<td>( \beta_1 )</td>
<td>(.3493) (.0991)</td>
<td>(.2537) (.1543)</td>
</tr>
<tr>
<td>( \beta_2 )</td>
<td></td>
<td>(.1834) (.1371)</td>
</tr>
</tbody>
</table>

Notes: (1) The figures in parentheses are posterior standard deviations.
(2) \( \rho = \max_{i=1,\ldots,p} \text{abs}(\text{root}_i) \), \text{root}_i=inverse of the \( i \)-th root of \( \Phi(B) \).
Table 5: Posterior Means and Standard Deviations: Baht/US$, Using CPI

<table>
<thead>
<tr>
<th></th>
<th>Untruncated ARMA(2,1)–GARCH(1,1)</th>
<th>Doubly Truncated ARMA(1,1)–GARCH(1,3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_1$</td>
<td>3.2785 (.1890)</td>
<td>3.2724 (.0685)</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>.1890 (.3460)</td>
<td>1.2933 (.5801)</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>1.6399 (.6877)</td>
<td>.8326 (.0693)</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-.6694 (.6877)</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>1.0297 (.0685)</td>
<td>.8326 (.0693)</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>-.6227 (.5890)</td>
<td>.0921 (.2606)</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>.00040 (.0046)</td>
<td>.00083 (.00581)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>.8229 (.1206)</td>
<td>.7006 (.1718)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>.3924 (.0815)</td>
<td>.2400 (.1224)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td></td>
<td>.0949 (.0856)</td>
</tr>
<tr>
<td>$\beta_3$</td>
<td></td>
<td>.0756 (.0753)</td>
</tr>
</tbody>
</table>

Notes: (1) The figures in parentheses are posterior standard deviations. 
(2) $\rho = \max_{i=1,\ldots,p} (\text{abs(root}_i))$, root$_i$=inverse of the $i$-th root of $\Phi(B)$. 
Table 6: Posterior Means and Standard Deviations: Won/US$, Using WPI

<table>
<thead>
<tr>
<th></th>
<th>Untruncated ARMA(1,1)–GARCH(1,2)</th>
<th>Doubly Truncated ARMA(1,1)–GARCH(2,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_1$</td>
<td>6.5862 (.1187)</td>
<td>6.5219 (.2269)</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>.7998 (.1632)</td>
<td>.7560 (.3532)</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>.9562 (.0316)</td>
<td>.9576 (.0629)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>.9562 (.0316)</td>
<td>.9576 (.0629)</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>.2101 (.1102)</td>
<td>.4637 (.3089)</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>.00188 (.01070)</td>
<td>.00214 (.01101)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>.6352 (.1270)</td>
<td>.7221 (.2285)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td></td>
<td>.4736 (.3887)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>.2862 (.1371)</td>
<td>.3253 (.2057)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>.2641 (.1099)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: (1) The figures in parentheses are posterior standard deviations.
(2) $\rho = \max_{i=1,\ldots,p} \{\text{abs(root}_i\} \text{, root}_i=\text{inverse of the i-th root of } \Phi(B)$.}
Table 7: Posterior Means and Standard Deviations: Won/US$, Using CPI

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Untruncated ARMA(1,1)–GARCH(1,2)</th>
<th>Doubly Truncated ARMA(1,1)–GARCH(1,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_1$</td>
<td>6.5849 (..1192)</td>
<td>6.6851 (.4929)</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>.7904 (.1577)</td>
<td>.8233 (.3920)</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>.9354 (.0346)</td>
<td>.9117 (.1262)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>.9354 (.0346)</td>
<td>.9117 (.1262)</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>.4789 (.0849)</td>
<td>.4580 (.1538)</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>.0016 (.0106)</td>
<td>.0023 (.0107)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>.9522 (.1144)</td>
<td>.8474 (.2527)</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>.3107 (.1141)</td>
<td>.2921 (.1821)</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>.0447 (.0478)</td>
<td>.1434 (.1234)</td>
</tr>
</tbody>
</table>

Notes: (1) The figures in parentheses are posterior standard deviations. 
(2) $\rho = \max_{i=1,\ldots,p} (\text{abs}(\text{root}_i))$, root$_i$ = inverse of the $i$-th root of $\Phi(B)$.

5 Concluding Remarks

We tested whether PPP holds for the Baht/US$ and the Won/US$ exchange rates using a doubly truncated regression model with an ARMA-GARCH error process. We employed the MCMC algorithms. We find that PPP is supported when we incorporate the fact that these currencies had been managed to stay within bounds.
We chose the Baht and Won exchange rates since they were the major currencies that were involved in the Asian financial crisis which started in Thailand in July 1997 and spread to Korea in November 1997. There are those who say that the overvaluation of these currencies is the underlying cause of the Asian financial crisis. Chinn (2000) and Sazanami and Yoshimura (1999), for example, measure overvaluation assuming that the exchange rates are determined by PPP. In this paper we verified that PPP holds when we use the doubly truncated regression model.
References

Ahking, F.W., 1997 Testing long-run purchasing power parity with a Bayesian unit root approach: the experience of Canada in the 1950s, Applied Economics, 29, 813–819


Bank of Japan, 1983 *Kawase rate kettei mechanism no bunseki (sono ichi)* (analysis of the mechanism of the exchange rate determination, Part 1), May 1983 mimeograph


Chinn, M.D., 2000 Three measures of East Asian currency overvaluation, Contemporary Economic Policy, 18:2, April, 205–214


Goldfajn, I. and R. Valdes, 1999 The aftermath of appreciation, Quarterly Journal of Economics, 114, 229–262

25

Goldman, E. and H. Tsurumi, 2001 Bayesian analysis of a doubly truncated ARMA-GARCH regression model with applications, mimeograph

Henricsson, R., E. Lundbäck, 1995 Testing the presence and the absence of purchasing power parity: results for fixed and flexible regimes, Applied Economics, 27, 635–641

Hoque, A., 1995 A test of the purchasing power parity hypothesis, Applied Economics, 27, 311–315


Lothian, J.R., 1997 Multi-country evidence on the behavior of purchasing power parity under the current float, Journal of International Money and Finance, 16, 19–35


Officer, L.H., 1982 Purchasing power parity and exchange rates: theory, evidence and relevance, (JAI Press, Greenwich, Connecticut)

Olekalns, N. and N. Wilkins, 1998, Re-examining the evidence for long-run purchasing power parity, the Economic Record, 74, 54–61


Rogoff, K., 1996 The purchasing power parity puzzle, Journal of Economic Literature, 34, 647–668


Sercu, P., R. Uppal and C. Van Hulle, 1995 The exchange rate in the presence of transaction costs: implications for tests of purchasing power parity, the Journal of Finance, 50, 1309–1319

Snell, A., 1996 A test of purchasing power parity based on the largest principal component of real exchange rates of the main OECD economies, Economics Letters, 51, 225–231


Valieva, E. and H. Tsurumi, 2001 A Bayesian convergence criterion for MCMC draws, mimeograph


Appendix: MCMC Algorithms

Since the detailed discussion of the MCMC algorithms for the doubly truncated ARMA\((p, q)\)-GARCH\((r, s)\) regression model is given in Goldman and Tsurumi (2001), let us present salient features of the algorithms. The model is given by

\[
y_t = x_t \gamma + u_t
\]

\[
u_t = \sum_{j=1}^{p} \phi_j u_{t-j} + \epsilon_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j}
\]

\[
\sigma_t^2 = \alpha_0 + \sum_{j=1}^{r} \alpha_j \epsilon_{t-j}^2 + \sum_{j=1}^{s} \beta_j \sigma_{t-j}^2
\]

\[
\alpha_0 > 0, \quad \alpha_j \geq 0, \quad j = 1, \ldots, r, \quad \beta_j \geq 0, \quad j = 1, \ldots, s
\]

and \(\epsilon_t \sim N(0, \sigma_t^2)\). The regressand \(y_t\) is doubly truncated:

\[
L_{\ell t} \leq y_t \leq L_{ut}
\]

where \(L_{\ell t}\) and \(L_{ut}\) are lower and upper bounds, respectively. The posterior density of the model is

\[
p(\delta | Y, X) = \frac{\ell(Y | X, \delta) p(\delta)}{\int \ell(Y | X, \delta) p(\delta) d\delta}
\]

where \(\delta\) is the set of all parameters in the ARMA–GARCH model; \(\ell(Y | X, \delta)\) is the likelihood function; \(Y\) is the \(n \times 1\) vector of \(y_t\)’s; \(X\) is the \(n \times k\) matrix of observations on the regressors \(x_t\), and \(p(\delta)\) is the prior pdf. The likelihood function is given by

\[
\ell(Y | X, \delta) \propto \prod_{t=1}^{n} \left\{ \frac{1}{\sigma_t} \exp \left( -\frac{\epsilon_t^2}{2\sigma_t^2} \right) \right\}
\]

where

\[
a_t = \frac{L_{\ell t} - g(Z_t)}{\sigma_t}, \quad b_t = \frac{L_{ut} - g(Z_t)}{\sigma_t}
\]

\[
g(Z_t) = x_t \gamma - \sum_{j=1}^{p} \phi_j \epsilon_{t-j} - \sum_{j=1}^{q} \epsilon_{t-j}
\]
\( \epsilon_t = y_t - g(Z_t) \)

\( \epsilon_t = y_t - x_t \gamma \)

We assume that \( y_0 = \epsilon_0, y_t = 0 \), for \( t < 0 \), and \( x_t = 0 \) for \( t \leq 0 \). We set the pre-sample error \( \epsilon_0 \) to be zero. As the prior pdf we use the following normal distributions:

\[
p(\epsilon_0, \gamma, \phi, \theta, \alpha, \beta) = N(\mu_{\epsilon_0}, \sigma^2_{\epsilon_0}) \times N(\mu_{\gamma}, \Sigma_{\gamma}) \times N(\mu_{\phi}, \Sigma_{\phi}) \times N(\mu_{\theta}, \Sigma_{\theta}) \times N(\mu_{\alpha}, \Sigma_{\alpha}) \times N(\mu_{\beta}, \Sigma_{\beta})
\]

(14)

We use the Metropolis-Hastings (MH) algorithms since we cannot generate samples from the posterior density directly. In the MH algorithm we draw \( \hat{\delta} \) from the proposal density \( g(\delta) \) and accept the drawn values \( \hat{\delta} \) with probability

\[
\lambda(\delta, \hat{\delta}) = \min \left\{ \frac{p(\hat{\delta}|Y, X)/g(\hat{\delta})}{p(\delta|Y, X)/g(\delta)}, 1 \right\}
\]

(15)

where \( g(\cdot) \) is the proposal density.

Following Nakatsuma (2000) we draw \( \gamma, \phi, \theta \) from the proposal densities that are based on the ARMA–GARCH model

\[
y_t = x_t \gamma + \sum_{j=1}^{p} \phi_j (y_{t-j} - x_{t-j} \gamma) + \epsilon_t + \sum_{j=1}^{q} \theta_j \epsilon_{t-j}
\]

(16)

where \( \epsilon_t \sim N(0, \sigma^2_t) \). We draw \( \alpha, \beta \) from the proposal densities based on the approximated GARCH model

\[
\epsilon_t^2 = \alpha_0 + \sum_{j=1}^{l} (\alpha_j + \beta_j) \epsilon_{t-j}^2 + w_t - \sum_{j=1}^{l} \beta_j w_{t-j},
\]

(17)

where \( w_t \sim N(0, 2\sigma_t^4) \) and \( l = \max\{r, s\} \); \( \alpha_j = 0 \) for \( j > r \) and \( \beta_j = 0 \) for \( j > s \).

The MCMC algorithms are divided into five blocks:
Block 1: regression coefficients, $\gamma = (\gamma_1, \cdots, \gamma_k)'$
Block 2: AR parameters, $\phi = (\phi_1, \cdots, \phi_p)'$
Block 3: MA parameters, $\theta = (\theta_1, \cdots, \theta_q)'$
Block 4: GARCH parameters, $\alpha = (\alpha_0, \cdots, \alpha_r)'$
Block 5: GARCH parameters, $\beta = (\beta_1, \cdots, \beta_s)'$

and we apply the MH procedure in each block.

The key difference between the MCMC algorithms for an untruncated ARMA–GARCH model and the MCMC algorithms for the doubly truncated ARMA–GARCH model is that we employ the random walk MH algorithms (Robert and Casella (1999), among others); we use the previous drawn values of $\delta$ rather than the maximum likelihood estimates (MLE) of $\delta$ as the means of the proposal densities. If we use the MLE of $\delta$ instead of the random walk then the posterior pdf of $\gamma_2$ becomes downward biased, but with the random walk draws we can avoid this.

Once the AR coefficients, $\phi_1, \cdots, \phi_p$, are drawn then we compute the $p$ roots of

$$\Phi(B) = 1 - \phi_1 B - \cdots - \phi_p B^p = 0$$

and change the roots to absolute values (in the case of a complex root it gets to be the complex conjugate), and we pick the maximum of the $p$ absolute values of the roots:

$$\rho = \max_{i=1,\cdots,p} \{\text{absolute values of the } p\text{-roots}\}.$$

In Goldman and Tsurumi (2001) it is suggested that the selection of an appropriate model should be based on the combination of measures: (i) the modified Bayesian information criterion (MBIC), (ii) the acceptance rate of each of the five blocks of the MH algorithms, and (iii) posterior standard deviations. The MBIC is given by

$$\text{MBIC} = -2 \ln m(x) + 2(k + p + q + r + s) \quad (18)$$

where $m(x)$ is the marginal likelihood of data, and $m(x)$ is computed by the harmonic mean suggested by Newton and Raftery (1994).

Table presents the MBIC and the acceptance rates of the MH algorithms in each of the five blocks (Blocks 1–5). The true model is $k = 2$, $p = q = r = s = 1$ with the level truncation given in the numerical example in the text.
Table 8: Modified Bayesian Information Criterion and MH Acceptance Rates for the Level Truncation Case

<table>
<thead>
<tr>
<th>Model</th>
<th>p</th>
<th>q</th>
<th>r</th>
<th>s</th>
<th>MBIC</th>
<th>MH acceptance rates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-.960.45*</td>
<td>Block 1  Block 2  Block 3  Block 4  Block 5</td>
</tr>
<tr>
<td>I</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-.957.46</td>
<td>.7305   .8605  .8368  .5568  .6304</td>
</tr>
<tr>
<td>II</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-.961.74**</td>
<td>.7427   .7991  .7991  .5818  .7455</td>
</tr>
<tr>
<td>III</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>-.953.07</td>
<td>.7427   .7991  .7991  .5818  .7455</td>
</tr>
<tr>
<td>IV</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>-.953.07</td>
<td>.7305   .8605  .8368  .5568  .6304</td>
</tr>
<tr>
<td>V</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>-.957.81</td>
<td>.7427   .7991  .7991  .5818  .7455</td>
</tr>
<tr>
<td>VI</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>-.954.24</td>
<td>.7427   .7991  .7991  .5818  .7455</td>
</tr>
<tr>
<td>VII</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>-.953.79</td>
<td>.7427   .7991  .7991  .5818  .7455</td>
</tr>
<tr>
<td>VIII</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>-.951.57</td>
<td>.7427   .7991  .7991  .5818  .7455</td>
</tr>
</tbody>
</table>

Notes: (1) The true model is Model I, sample size $n = 500$, level truncation.
(2) * indicates the second smallest MBIC; ** indicates the smallest MBIC.
(3) Block 1: regression coefficients,
    Block 2: AR parameters,
    Block 3: MA parameters,
    Block 4: GARCH parameters $\alpha$,
    Block 5: GARCH parameters $\beta$

The true model is Model I, and the MBIC for this model is a close second to the MBIC of Model III in which the order of the MA process is misspecified to be 2 rather than 1. This shows that the MBIC (or any other criteria such as the Bayes factors or posterior odds ratios) does not always pick the correct model, and thus it is not wise to rely on a single criterion. The acceptance rates for each of the five blocks of the MH algorithms show that those blocks in which the number of parameters are chosen to be greater than the true number tend to have lower acceptance rates. For example in model II, $p$, the AR order, is set to be 2 ($p = 2$) and the MH acceptance rate of Block 2 (AR parameters) is .7991, whereas in model I (the true model) the MH acceptance for Block 2 is .8605. Although it is not shown in Table 8, we find that the posterior standard deviations tend to be larger when the number of parameters is misspecified. Hence, the model selection should be made on the combination of the MBIC, MH acceptance rates, and posterior standard deviations as well as the convergence patterns and autocorrelations of the draws.
The convergence of the MCMC draws is judged by the Bayesian modification of the test of the constancy of the regression parameters proposed by Nabeya and Tanaka (1988). This modification is given in Valieva and Tsurumi (2001). This Bayesian Nabeya and Tanaka test gives more information about the convergence than the usual convergence criteria such as those Kolmogorov-Smirnov statistic or CUSUM test which are explained in Cowles and Carlin (1996).