SUMMARY OF RECENT ACHIEVEMENTS

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The n-dimensional torus T^n is the classical compact abelian Lie group, i.e., the product of n circles. The study of topological spaces with torus actions has become increasingly important in various areas of mathematics. Recently, a field of activity has appeared with the title toric topology. The classical model is provided by a toric manifold, a compact non-singular toric variety. A toric variety has an algebraic torus $(\mathbb{C}*)^n$ action with a dense orbit. We regard the compact torus T as the standard subgroup in algebraic torus. Then the orbit space of a toric manifold with torus action can be identified with the simple polytope and the action of T is locally standard, that is, locally modeled by the standard action on $\mathbb{C}^n$. Davis and Januszkiewicz first introduced the notion of a topological generalization of a toric manifold, which is now called a quasitoric manifold, by taking these two characteristic properties as the starting point. A quasitoric manifold is a smooth closed manifold with locally standard half-dimensional torus action which has a simple polytope as an orbit space. Similarly, they also defined a small cover which is a real-version of a quasitoric manifold, i.e., a closed manifold with a locally standard real torus action whose orbit space is simple. My major research area is the toric topology. Fortunately, some results have been developed in my work, partly joint with my advisor, Dong Youp Suh, and other colleagues.

Quasitoric manifolds over a product of simplices

One of the most interesting examples of (quasi-) toric manifolds is a Hirzebruch surface. If B is a toric manifold and E is a Whitney sum of complex line bundles over B, then the projectivization $P(E)$ of E is again a toric manifold. Starting with B as a point and repeating this construction, we obtain a sequence of complex projective bundles which we call a generalized Bott tower. In fact, the top manifold in tower, called a Bott manifold, is indeed a (quasi-) toric manifold and its orbit space can be identified with a product of simplices. Similarly, when E is a Whitney sum of real line bundles, we get a sequence of real projective bundles, which is called a real generalized Bott tower. It also provides an example of small covers over a product of simplices. It turns out that every small cover over a product of simplices is equivalent (in the sense of Davis and Januszkiewicz) to a generalized real Bott tower, but this is not the case for quasitoric manifolds. We show that a quasitoric manifold over a product of simplices is equivalent to a generalized Bott tower if and only if it admits an almost complex structure left invariant under the action. Finally, we show that a quasitoric manifold

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over a product of simplices is homeomorphic to a generalized Bott manifold if it has the same cohomology ring as a product of complex projective spaces.

Topological classification of generalized Bott tower
The topological classification of these manifolds has recently attracted the attention of toric topologists. For instance, it is well-known that Hirzebruch surfaces can be distinguished by their cohomology rings up to diffeomorphism. Motivated by this, of special interest is the following problem which is now called the cohomological rigidity problem; Are toric manifolds diffeomorphic (or homeomorphic) if their cohomology rings are isomorphic as graded rings? In this paper, we give partial affirmative solutions to this problem. Note that the class of Bott manifolds is a generalization of the class of Hirzebruch surfaces. We prove that if the Bott manifold has the same cohomology ring as a product of complex projective spaces, then every fibration in the tower is trivial so that the Bott manifold is diffeomorphic to the product of complex projective spaces. Combining with the above result, if any quasitoric manifold over a product of simplices whose cohomology ring is isomorphic to the cohomology ring of the product of projective spaces, then they are homeomorphic. Moreover, we show classes of 2-staged generalized Bott manifolds and 3-staged Bott manifolds also give supporting evidences to the cohomological rigidity problem.

Toric cohomological rigidity of simple polytopes
A simple polytope P is (toric) cohomologically rigid if its combinatorial structure is determined by the cohomology ring of a quasitoric manifold M over P, i.e., there exists a quasitoric manifold M over P, and whenever there exists a quasitoric manifold N over another polytope Q with $H^*(M) = H^*(N)$ there is a combinatorial equivalence $PapproxQ$. Although $H^*(M)$ contains some information of P, not every simple polytope has this property, but some important polytopes such as simplices or cubes are known to be cohomologically rigid. In this paper, we investigate the cohomological rigidity of polytopes and establish it for several new classes of polytopes including products of simplices. The main idea is that the ring isomorphism between the cohomology rings implies the algebra isomorphism between the Tor-algebra of P and Q and hence a cohomological rigidity is related to the bigraded Betti numbers of its Stanley-Reisner ring, another important invariants coming from combinatorial commutative algebra.

On the other hand, I do research about the small covers over a cube. The class of these manifolds is a very interesting subject because it has a nice connection between linear algebra and topology. One of my interests in the category is the enumeration.

The number of small covers over cubes
In this paper, we find a bijection between the set of small covers over an n-cube and the set of acyclic digraphs with n labeled nodes. Using this, we establish the formula of the number of small covers over cubes and products of simplices in the sense of Davis and Januszkiewicz. Moreover we

count the equivariant homeomorphism classes and prove that the number of acyclic digraphs with n unlabeled nodes is an upper bound of the number of small covers over an n-cube up to diffeomorphism. This work was presented as a poster at a conference, New Horizons in Toric Topology, July 7-11, 2008, University of Manchester, UK, which is the biggest conference in Toric Topology. In this conference, this work won a prize as a best presentation.

References

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